

Anisotropy as a consequence of pre-equilibrium rescattering

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We show that azimuthal asymmetry in transverse momentum distribution of particles observed in non-central heavy ion collisions can originate from pre-equilibrium rescattering. The phenomenon is studied using computer simulation of expanding pion gas created in Pb+Pb 160 GeV/n collisions. Results obtained are discussed in comparison with experimental results of NA49 collaboration. Theoretical understanding of mechanism generating the asymmetry is presented. Conclusion about non-equilibrium properties of the phenomenon studied is drawn.

I. INTRODUCTION

Since the discovery of directed flow in pioneering experiments on Bevalac [1] the phenomenon of azimuthal asymmetries has been studied extensively at various beam energies using different types of colliding nuclei. Besides the interesting phenomenon of directed flow two additional signatures of the collective behavior of hadrons have been identified in heavy ion collisions (HIC):

1) *Radial flow*, existence of which has been clearly established in central collisions [2]. Recently signature of radial flow in *non-central* collisions has been predicted [3] and most likely also confirmed experimentally [4].

2) *Elliptic flow*, which can be oriented orthogonally to the reaction plane (squeeze-out) at Bevalac energies [5] or parallel to the reaction plane at AGS [6,7] or SPS [4,11] energies. In Ni+Ni 2GeV/n experiments orientation of the elliptic flow depends on transverse momentum region of neutral mesons selected for the analysis [8].

Phenomenon of azimuthal asymmetries is satisfactorily understood as a consequence of collective hydrodynamical behavior of nuclear matter described by the equation of state or as a result of shadowing effect and absorption processes in spectator parts of colliding nuclei.

In this work we study properties of second-order elliptic flow of final state pions. We concentrate mainly on anisotropy created in Pb+Pb 160 GeV/n non-central collisions studied recently by NA49 collaboration using Main TPC [4] and Ring Calorimeter [9] data.

Second-order flow oriented parallel to the reaction plane was predicted in theoretical work of J.-Y. Ollitrault [10] for ultra-relativistic energy domain. Mechanism of in-plane elliptic flow generation [10] is based on the assumption of thermalization of nuclear matter in the

asymmetrical overlapping region of non-centrally colliding nuclei. This assumption is essential for the existence of different pressure gradients in different directions inside the asymmetrical overlapping region. During the subsequent hydrodynamical evolution different pressure gradients generate second-order asymmetry in transverse momentum distribution of particles [10]. Also in the recent theoretical work on elliptic flow at AGS energies [7] an assumption about weak sensitivity of the resulting flow asymmetry to non-equilibrium effects (in central rapidity region) is formulated.

In this work we show that elliptic flow can be generated by rescattering process among produced hadrons (pions) without the equilibration in prehadronic or hadronic stage of heavy ion collision. Our study is motivated by results of computer simulation [11] of the rescattering process in the expanding pion gas created in Pb+Pb 160 GeV/n non-central collisions.

The paper is organized as follows: In section II we briefly describe our rescattering simulation of the expanding pion gas - the CASCUS 1.0 model. Main results of the simulation - centrality dependence, p_t dependence, rapidity dependence and the equilibration dependence of the asymmetry are presented in section III. The results are compared with experimental data [4]. Qualitative arguments offering our understanding of the mechanism generating the asymmetry are presented in Section IV. At the end of this work a summary of results is given and short conclusions are drawn.

II. RESCATTERING PROCESS IN PION GAS

Total number of pions generated in ultrarelativistic Pb+Pb collisions at SPS energy is surprisingly large. Even more than 2500 pions can be generated in a central Pb+Pb collision at this energy. Number of secondary particles thus exceeds number of primary nucleons in the system. Assuming that pions are created in independent nucleon-nucleon collisions the initial transverse momentum and rapidity spectra of produced pions are determined mainly by the dynamics of pion production in nucleon-nucleon collisions ¹.

¹Even if this picture of pion formation may be oversimplified we believe that it can provide a reasonable approximation to reality for SPS energy region

Our rescattering simulation is based on the following scenario: Initial state of the pion gas is characterized by momentum and spatial distribution $\rho(\vec{p}, \vec{x}, t)$ of pions as they are produced in nucleon-nucleon collisions. As the system evolves positions of pions change naturally due to the classical motion of pions. Momenta of pions can change as well in mutual collisions of pions. As the size of the cloud of pions increases, spatial density of pions becomes smaller and consequently collisions among pions become rare. At some point collisions cease at all and momentum distribution of pions is not influenced any more. Pions move then freely on their way to detectors. Thus momentum distribution of pions recorded by detectors is not equal to the original momentum distribution of pions as they were produced in nucleon-nucleon interactions. It is changed during rescattering process in the early stage of the pion gas expansion. In next subsection we give short description of the computer simulation we have performed.

A. Simulation of rescattering process

Our rescattering model [11] is similar to the rescattering simulation described by T.J.Humanic [12]. Two additional concepts have been used in our version of rescattering model:

- 1) Initial state of the pion gas in non-central collisions was generated by cascade program [13] and
- 2) Formation time of hadrons [14] was introduced. The simulation is performed in CMS of the Pb+Pb collision with time step $\Delta T = 0.1 fm/c$. In each time step positions of pions are changed according to the classical equation of motion $\vec{x}(T + \Delta T) = \vec{x}(T) + \vec{v} \cdot \Delta T$. Two pions collide when their distance is smaller than critical distance $d_c(s) = \sqrt{\sigma(s)/\pi}$ where $\sigma(s)$ is isospin averaged elastic pion-pion cross section (taken from experimental data [15]). New momenta of pions are determined in CMS frame of pion-pion collision using differential cross section of the elastic pion-pion interaction. Then new momenta of pions are transformed back to the global frame of the simulation (CMS of Pb+Pb system).

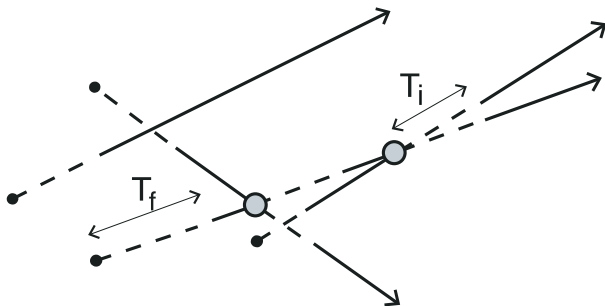


Fig.1 Time delay parameters incorporated in our rescattering simulation.

After being created pions are not allowed to participate in mutual interactions for Lorentz dilated time interval - the formation time $T_f = \gamma \cdot \tau_f$ and for interaction delay time T_i after each collision. Formation time τ_f influences expansion of the pion gas mainly in the initial stage of time evolution, interaction time T_i suppresses unphysical effects of rescattering simulations [16] and allows to influence total number of collisions in the pion gas.

Since no collective behavior of primary nucleons is incorporated in cascade generator [13] and since the production of pions in primary nucleon-nucleon collisions is azimuthally isotropic the initial state of the pion gas is azimuthally isotropic in momentum space also in the case of non-central collisions:

$$\varrho(\vec{x}, \vec{p}, t) = \chi(t) \cdot A^T(\vec{x}) \cdot S^T(\vec{p}) \quad (1)$$

Here A^T denotes azimuthally asymmetrical and S^T denotes azimuthally symmetrical distribution in transverse plane. During the pion gas expansion symmetrical $S^T(\vec{p})$ distribution is changed by mutual collisions of pions.

First results of our simulation (centrality dependence of the asymmetry) have been presented during Heavy Ion Workshop on Particle Physics (September 1996) in Slovakia [11,17,18]. Detailed description of the simulation can be found in thesis [19]. In following subsections we present additional results of the rather extensive simulation we have performed.

III. RESULTS OF THE SIMULATION

Rescattering simulation was performed for random and unknown orientations of the reaction plane in events generated by cascade program [13]. During the subsequent analysis impact parameter orientation had been determined for each event [17] and histograms of azimuthal distributions of pions had been filled (see Fig.2). Azimuthal asymmetry of second order oriented in the reaction plane (not present in the initial state of the pion gas) had been clearly identified in transverse momentum distribution of pions after the rescattering process.

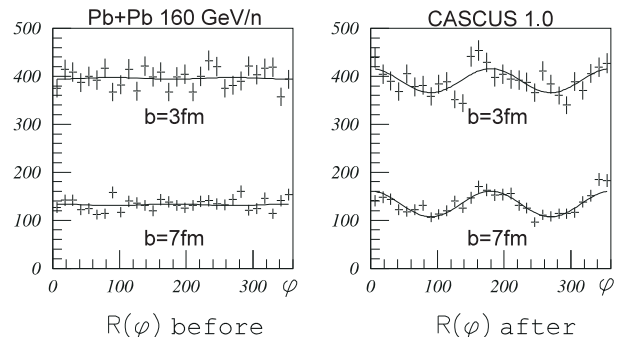


Fig.2 Azimuthal distribution of pions before and after the rescattering process for events with $b = 3fm$ and $7fm$.

Strength of the asymmetry was evaluated by fit to the function:

$$R(\phi) = S_o[1 + S_2 \cdot \cos(2\phi)] \quad (2)$$

In next subsections we present main properties of the asymmetry found.

A. Centrality dependence

Events with impact parameter $b = 3, 5, 7, 9, 10, 11, 12 \text{ fm}$ generated by cascade program [13] have been used as input for the rescattering program and subsequently analyzed for second-order asymmetry. Following behavior had been found for different values of rescattering model parameters τ_f, T_i :

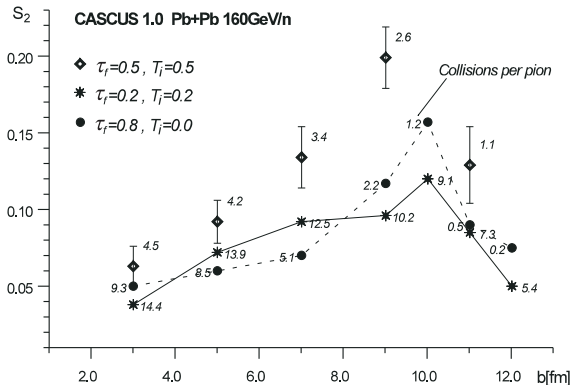


Fig.3 Centrality dependence of the asymmetry for different values of τ_f, T_i parameters. Number of collisions per pion (indicated for each data point) decreases with increasing value of impact parameter. Non-zero asymmetry is generated also for small number of collision per pion (0.5 Coll/pion at $b=11 \text{ fm}$).

Asymmetry increases with impact parameter up to the region $b = 9-10 \text{ fm}$ and then it decreases. This behavior has been predicted in the work of J.-Y. Ollitrault [10] and it is in remarkable agreement with experimental data [9]. From the point of view of rescattering simulation centrality dependence of the asymmetry can be understood easily. For $b = 0$ initial conditions have cylindrical symmetry and there is therefore no reason for the asymmetry in the final state. For very peripheral collisions interactions of pions are very rare due to low multiplicity of pions. Therefore asymmetry is expected to decrease for peripheral collisions. Sizeable asymmetry is generated in the intermediate region where spatial asymmetry in the initial state and number of pion-pion collisions are sufficiently large.

Shape of the centrality dependence is not very sensitive to the number of collisions per pion in the expanding pion gas (see Fig.3).

We find also to be surprising, that the asymmetry is generated in the case of small number of collisions per pion (e.g 0.5 collision per pion). Dependence of the strength of asymmetry on the equilibration of the pion gas is studied at the end of this section. Now we present results on p_t and rapidity dependence of the asymmetry.

B. Transverse momentum dependence

Transverse momentum dependence of the asymmetry has been identified in our results already in the early stage of this work [11]. It has been studied quantitatively for different values of rescattering program parameters τ_f, T_i . 100 events with impact parameter $b = 7 \text{ fm}$ have been used for this analysis. Selected results are presented in Fig.4.

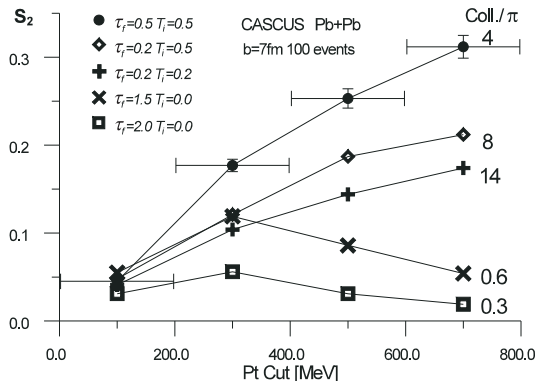


Fig.4 p_t dependence of the elliptic flow of pions for different values of parameters τ_f, T_i .

The asymmetry increases with p_t for number of collisions per pion $N_{coll./\pi} = 4, 8, 14$ in rough agreement with experimental data [4]. For $N_{coll./\pi} = 0.6, 0.3$ the shape of the p_t dependence changes together with absolute strength of the asymmetry. Decrease of the asymmetry strength at large p_t for $N_{coll./\pi} = 0.3, 0.6$ can be understood easily. High p_t pions can escape from the pion gas volume without interaction due to large value of Lorentz-dilated formation time T_f and also due to their larger velocity. Therefore smaller asymmetry is generated for rarely colliding high- p_t pions while larger asymmetry is generated for colliding low- p_t pions. Decrease of the asymmetry in case $N_{coll./\pi} = 8, 14$ in comparison with $N_{coll./\pi} = 4$ is discussed in section D.

Detailed comparison of p_t dependence of asymmetry with experimental data may allow to determine physical parameters incorporated in the simulation (in particular the formation time).

For this goal however improvements in our rescattering model (e.g. dynamics of T_i delay parameter) and also higher statistics of experimental data [4] would be necessary.

C. Rapidity dependence

Rapidity dependence of the asymmetry has been studied using $b = 7fm$ events sample. Resulting rapidity dependence found by our simulation exhibits maximum at distance $\Delta Y = 0.75$ from central rapidity region. We find to be interesting that results on rapidity dependence of elliptic flow presented during Quark Matter 97 conference [4] also exhibit decrease of the asymmetry strength at mid-rapidity region.

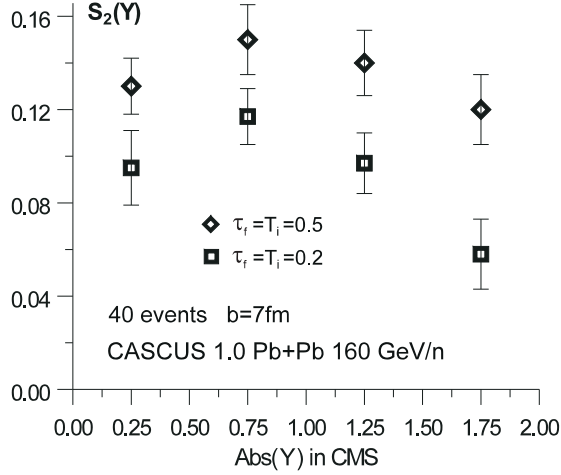


Fig.5 Rapidity dependence of the asymmetry. Note decrease of the asymmetry at midrapidity.

This result is worth of further investigation. It is not excluded that the origin of the mid-rapidity decrease of the asymmetry is related to behavior studied in the next subsection.

D. Thermalization dependence

Numerical value of the rescattering model parameters (see Fig.1) allows to influence total number of collisions in the expanding pion gas. We have studied dependence of the asymmetry on total number of collisions in the pion gas for the set of $b = 7fm$ events. Following result has been obtained (Fig.6):

With increasing number of collisions in the pion gas the asymmetry first increases, it exhibits maximum close to $N_{coll}/\pi = 4$ region and then it decreases. This decrease depends mainly on number of collisions per pion, it is not a consequence of the physics of formation time τ_f or interaction time T_i parameter. Both $\tau_f = 0.5, T_i = 0.2$ and $\tau_f = 0.2, T_i = 0.5$ combinations lead to decrease of asymmetry from $\tau_f = 0.5, T_i = 0.5$ level (see Fig.6). This result can be interpreted as a non-equilibrium feature of the mechanism generating the asymmetry. We think that monotonic increase and saturation should be expected if the asymmetry were generated in an equilibrium process.

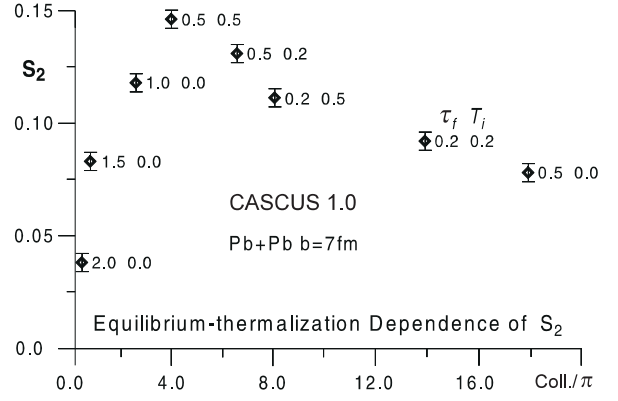


Fig.6 Thermalization dependence of the asymmetry.

For high number of collision per pion $N_{coll.}/\pi \approx 16$ the asymmetry seems to saturate. We guess this corresponds to hydrodynamical limit studied in work [10].

At this point we can try to understand the decrease of asymmetry at mid-rapidity found in our simulation (Fig.5) and present also in experimental data [4]. The decrease can be explained as a consequence of smaller number of collisions per pion ($N_{coll.}/\pi < 4$) at midrapidity in comparison with number of collisions per pion at the maximum of rapidity dependence². This point requires further and more careful study since the position of the maximum is different in our results and experimental data [4].

E. Time evolution of the asymmetry

Numerical simulation of the pion gas expansion allows us to study also quantities which cannot be accessed from final stage of the pion gas. We have studied time evolution of the number of collisions in the pion gas together with the time evolution of second-order asymmetry. For the sake of simplicity the asymmetry has been characterized by R_p parameter:

$$R_p = \frac{\langle p_y^2 \rangle}{\langle p_x^2 \rangle} \quad (3)$$

which does not require the fit procedure for its evaluation.

From results shown in Fig.7 we conclude that the asymmetry is generated during the early stage of the expansion. This confirms our conjecture that the asymmetry can originate during pre-equilibrium stage of the pion gas expansion. Theoretical understanding of mechanism the asymmetry is generated by is presented in next section.

²This explanation is supported by the simulation of dilepton production from pion gas [20].

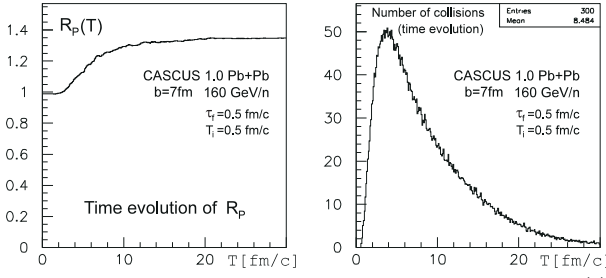


Fig.7 Time evolution of the symmetry parameter $R_p(t)$ and evolution of number of collisions during the expansion of pion gas. Time dependence of $R_p(T)$ is nearly identical with the behavior of $S_2(T)$ parameter [19].

IV. THEORETICAL ANALYSIS OF ASYMMETRY

Expansion of the interacting pion gas can be described by kinetic equation [21] which (in the case of zero external macroscopic field) takes the form:

$$\frac{\partial f(\vec{p}, \vec{x}, t)}{\partial t} + \vec{v}_p \cdot \nabla_{\vec{x}} f(\vec{p}, \vec{x}, t) = [St]_{Coll} \quad (4)$$

$[St]_{Coll}$ is the collision (or scattering) term describing influence of pion-pion collisions on distribution function $f(\vec{p}, \vec{x}, t)$.

In slightly simplified case (we do not take into account time interval during which pions are produced) our initial state of the pion gas - Eq.(1) can be written in the form:

$$\rho(\vec{x}, \vec{p}, T_0) = A^T(\vec{x}_t, x_z) \cdot S^T(\vec{p}_t, p_z) \quad (5)$$

where again $A^T(\vec{x}_t, x_z)$ denotes asymmetrical spatial distribution of pions in transverse plane and $S^T(\vec{p}_t, p_z)$ is symmetrical distribution of pions in transverse momentum. During the process of pion gas expansion asymmetry in spatial distribution of pions "leaks" into momentum distribution of pions which becomes asymmetrical in transverse momentum space.

This phenomenon results from mutual interactions of pions in the expanding asymmetrical volume. If there were no collisions of pions, momentum distribution of pion gas (momenta of pions) would be unchanged. This is clear intuitively and it can be seen also on analytical level after the integration of kinetic equation (4) in spatial coordinates and after the subsequent time integration:

$$F_A(\vec{p}_t, p_z, T) = F_S(\vec{p}_t, p_z, T_0) + \int_{T_0}^T \int [St] d^3x dt \quad (6)$$

Here distribution $F(\vec{p}, t) = \int F(\vec{p}, \vec{x}, t) d^3x$ is distribution of pions in momentum space. Since original momentum distribution of pions $F_S(\vec{p}, T_0)$ is symmetrical in \vec{p}_t plane (our initial condition) and since resulting distribution $F_A(\vec{p}, T)$ is asymmetrical (this distribution is measured by detectors) the collision integral $\int_{T_0}^T \int [St] d^3x dt$ must be asymmetrical.

Asymmetry of collision integral in Eq.(6) originates from the asymmetrical spatial distribution of pions in transverse plane. We do not attempt to evaluate the collision integral here though we guess this is accomplishable task. The asymmetry of the collision integral can be understood intuitively using Toy model of pion gas expansion described in the previous paper [11]. In next section we derive formalism of two point decomposition of source which (we think) allows to study the asymmetry theoretically.

A. Two point source of pions.

Let us consider two point-like sources of pions separated by distance D in transverse plane. Each source radiates pions with random azimuthal angle and (for simplicity) with fixed absolute value of momentum. If we switch off rescattering of pions emitted from these two sources azimuthal distribution of pions observed in a distant detector surrounding the source ($D \ll R$) is constant - symmetrical. However if collisions of pions are allowed the resulting distribution of pions is asymmetrical in transverse momentum plane. Excess of pions in direction orthogonal to \vec{D} is generated as a consequence of collisions (see Fig.8).

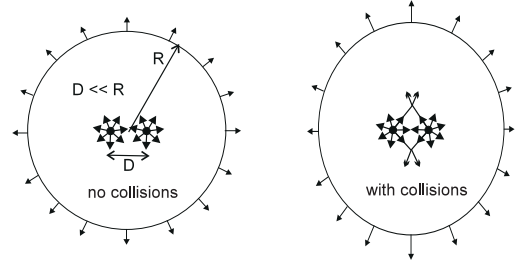


Fig.8 Azimuthal asymmetry generated by interactions of pions emitted from two point-like sources of pions.

Elliptic asymmetry can be characterized by R_p parameter - Eq.(3) and therefore we shall express $p_{y_1'}^2 + p_{y_2'}^2$ quantity for a given pair of pions after the collision at any place of transverse plane. If $p_{y_1'}^2 + p_{y_2'}^2$ has increased then also R_p parameter increases what means that elliptic asymmetry is generated. After Lorentz boosting back from CMS of $\pi\pi$ collision to the rest frame of two-point source and after averaging over random orientations of pion momenta after the collision (in CMS of $\pi\pi$ collision) we obtain the result (see Appendix):

$$\langle p_{y_1'}^2 + p_{y_2'}^2 \rangle_\alpha = p_{CMS}^2 + \gamma^2 \beta_y^2 \cdot \Lambda(p_{CMS}, \beta) \quad (7)$$

where $\Lambda(p_{CMS}, \beta) = 2E_{CMS}^2 + p_{CMS}^2 \left[\frac{2}{\gamma+1} + \frac{\gamma^2 \beta^2}{(\gamma+1)^2} \right]$ and $\vec{\beta} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$. For $\langle p_{x_1'}^2 + p_{x_2'}^2 \rangle_\alpha$ the same expression can be found just β_y is replaced by β_x . Increase of R_p parameter is generated by positive values of

$$\Delta p_y^2 = \langle p_{y_1}^2 + p_{y_2}^2 \rangle_\alpha - p_{y_1}^2 - p_{y_2}^2 \quad (8)$$

quantity (p_{y_1}, p_{y_2} are momenta of pions before collision) and negative values of similarly defined Δp_x^2 quantity. Numerical values of Δp_y^2 and Δp_x^2 depend on coordinates of the collision point in transverse plane since $\vec{\beta}, p_{CMS}$ and also \vec{p}_1, \vec{p}_2 depend on coordinates of the collision point. In Fig.9 we show Mathematica [22] contour-plot of Δp_y^2 quantity for two point source radiating pions with equal ($p_1 = p_2$) and different ($p_1 > p_2$) momenta.

Two-point source represents (in rough approximation) overlapping region of colliding nuclei in transverse plane. It is clear, that resulting elliptic asymmetry is oriented in the direction of impact parameter - orthogonally to \vec{D} vector.

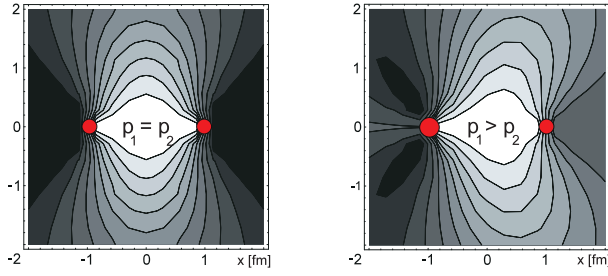


Fig.9 Contour plot of Δp_y^2 - see Eq.(8) for a two point source emitting pions with momenta p_1 and p_2 . White color represents positive values of Δp_y^2 quantity.

In next section we study asymmetry generated by non-trivial discrete and also continuous source.

B. Two-point source decomposition

Main idea of the two-point decomposition of source is following: Each spatially distributed source of interacting particles behaves (in 1 collision/pion approximation) as a set of two-point sources (see Fig.10). Each two-point source (characterized by vector \vec{D}) generates asymmetry with the orientation determined by vector \vec{D} . Our intention is to express analytically resulting asymmetry generated by the whole source.

Let $R_2(\phi, \vec{D})$ is azimuthal distribution of pions (particles) emitted from two-point source characterized by vector \vec{D} . Asymmetry of $R_2(\phi, \vec{D})$ is generated by collisions of pions at any place of transverse plane. Contribution from all possible collision points is averaged. Let us also have non-trivial e.g. discrete source radiating independent pairs of interacting pions. (This is our specification of 1 collision per pion approximation.)

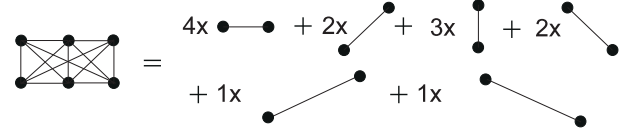


Fig.10 Discrete asymmetrical source of pions and its two-point decomposition. Note the excess of horizontally oriented two-point sources in comparison with vertical two-point sources.

Then in 1coll./ π approximation resulting azimuthal asymmetry of particles emitted from more complex but discrete source (see Fig.10) is given by weighted sum of azimuthal distributions of all possible two-point sources:

$$R_\rho(\phi) = \sum_{ij} W_{ij} \cdot R_2(\phi, \vec{D}_{ij}) \quad (9)$$

Weights W_{ij} depend on intensity of point-like sources i, j . Expression (9) can be extended to the case of spatially continuous source.

If we define probability distribution for the emission of a pair of particles from any places \vec{x}_1, \vec{x}_2 in the source $\rho(\vec{x})$ at relative distance $\vec{D} = \vec{x}_1 - \vec{x}_2$

$$W_\rho(\vec{D}) = \iint d^2x_1 d^2x_2 \left[\rho(\vec{x}_1) \rho(\vec{x}_2) \cdot \delta(\vec{D} - (\vec{x}_1 - \vec{x}_2)) \right] \quad (10)$$

what can be rewritten as

$$W_\rho(\vec{D}) = \int \rho\left(\frac{\vec{x} + \vec{D}}{2}\right) \cdot \rho\left(\frac{\vec{x} - \vec{D}}{2}\right) d^2\vec{x} \quad (11)$$

then resulting azimuthal distribution of particles emitted by spatially distributed source $\rho(\vec{x})$ can be expressed in 1 collision per pion approximation as follows:

$$R_\rho(\phi) = \int W_\rho(\vec{D}) \cdot R_2(\phi, \vec{D}) d^2\vec{D} \quad (12)$$

Dependence of the resulting $R_\rho(\phi)$ distribution on e.g. formation time and $\pi\pi$ cross section is encoded in distribution $R_2(\phi, \vec{D})$. If properties of $R_2(\phi, \vec{D})$ distribution are known, expression (12) allows us to say which source can generate asymmetry at analytical level - without numerical simulation.

Asymmetry of resulting R_ρ distribution is generated only by asymmetrical part of $W_\rho(\vec{D})$ distribution

$$A(D, \phi) = W_\rho(D, \phi) - S(D) \quad (13)$$

where $S(D)$ is symmetrical part of $W_\rho(D, \phi)$. $S(D)$ is equal to minimum of $W_\rho(D, \phi)$ at given D . Asymmetrical source distribution $\rho(\vec{x})$ leads to asymmetrical $W_\rho(\vec{D})$ distribution which results in non-zero asymmetrical distribution $A(D, \phi)$. Non-zero $A(D, \phi)$ distribution generates asymmetrical $R_\rho(\phi)$. Properties of $R_\rho(\phi)$ depend on analytical properties of $A(D, \phi)$ and $R_2(\phi, \vec{D})$.

V. SUMMARY AND CONCLUSIONS

We have shown that second order azimuthal asymmetry of hadrons known also as in-plane elliptic flow can be generated by rescattering process. We have performed extensive study of this asymmetry using computer simulation of Pb+Pb 160 GeV/n non-central collisions. Main properties of the asymmetry (impact parameter dependence, p_t dependence and rapidity dependence) studied by the simulation are in rough agreement with experimental data [4].

Transverse momentum dependence of the asymmetry (Fig.4) is found to be sensitive to formation time parameter. Careful analysis of experimental data together with improved simulation of hadron gas expansion might allow to determine experimental value of the formation time of pions from data.

Rapidity dependence of the second-order asymmetry of pions (Fig.5) exhibits minimum at midrapidity. Study of thermalization dependence of the asymmetry (Fig.6) allowed us to interpret this behavior as a consequence of different number of collisions among pions in different rapidity intervals.

Dependence of the strength of asymmetry on total number of collisions in the pion gas (Fig.6) together with non-zero value of resulting asymmetry in the $N_{coll}/\pi=0.5$ case (Fig.3) instigates us to conclude that the studied asymmetry can be generated in a non-equilibrium (pre-equilibrium) process. This conclusion substantially modifies our point of view on physical meaning of the in-plane elliptic flow of pions measured in experimental data.

We have developed formalism of two-point source decomposition (in one collision per pion approximation) which allows theoretical study of the asymmetry. We have shown that resulting distribution of pions in momentum space is related to the initial spatial distribution of pions.

We conclude that azimuthal anisotropy in momentum distribution of hadrons can be generated by pre-equilibrium rescattering process.

VI. APPENDIX

For a given pair of pions colliding at place (x, y) of transverse plane the absolute value of momenta in CMS frame of collision is:

$$p_{CMS} = \sqrt{M(x, y)^2/4 - m_\pi^2} \quad (14)$$

where $M(x, y)^2 = (E_1 + E_2)^2 - (\vec{p}_1(x, y) + \vec{p}_2(x, y))^2$. Let the orientation of momentum \vec{p}'_{CMS} after the collision in CMS of $\pi\pi$ collision is random and characterized by angle α : $p'_{x_1 CMS} = p_{CMS} \cdot \cos(\alpha)$; $p'_{y_1 CMS} = p_{CMS} \cdot \sin(\alpha)$. Then momentum of pion after the collision in the LAB frame (rest frame of two-point source) is:

$$\vec{p}'_1 = \vec{p}'_{1 CMS} + \gamma \vec{\beta}' \left(\frac{\gamma}{\gamma + 1} \vec{\beta}' \cdot \vec{p}'_{1 CMS} - E_{CMS} \right) \quad (15)$$

where $\vec{\beta}' = -\vec{\beta} = -\frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$. Consequently for e.g. y components of pion momenta after collision we have:

$$\begin{aligned} p'_{y_1} &= p'_{y_1 CMS} + \gamma \beta_y \cdot G_1 \\ p'_{y_2} &= p'_{y_2 CMS} + \gamma \beta_y \cdot G_2 \end{aligned} \quad (16)$$

where $G_1 = \frac{\gamma}{\gamma + 1} \vec{\beta}' \cdot \vec{p}'_{1 CMS} + E_{CMS}$ and similarly $G_2 = \frac{\gamma}{\gamma + 1} \vec{\beta}' \cdot \vec{p}'_{2 CMS} + E_{CMS}$. Then the sum $p'^2_{y_1} + p'^2_{y_2}$ gives:

$$p'^2_{y_1} + p'^2_{y_2} = 2p'^2_{y_1 CMS} + \gamma^2 \beta_y^2 G_{12}^{+2} + 2p'_{y_1 CMS} \gamma \beta_y G_{12}^- \quad (17)$$

where $G_{12}^- = G_1 - G_2$; $G_{12}^{+2} = G_1^2 + G_2^2$ and relationship $\vec{p}'_{1 CMS} = -\vec{p}'_{2 CMS}$ has been used. Equation (17) is valid for any given angle α of pion momentum after the collision in CMS. For the sake of simplicity we assume that this angle is random what allows to perform averaging of result (17) over values of α . This leads to the result:

$$\begin{aligned} \langle p'^2_{y_1} + p'^2_{y_2} \rangle_\alpha &= p_{CMS}^2 + \gamma^2 \beta_y^2 \left[\left(\frac{\gamma}{\gamma + 1} \right)^2 \beta^2 p_{CMS}^2 + 2E_{CMS}^2 \right] \\ &+ \frac{2\gamma^2}{\gamma + 1} p_{CMS}^2 \beta_y^2 \end{aligned} \quad (18)$$

where following sub-results have been used:

$$\langle G_1^2 + G_2^2 \rangle_\alpha = 2E_{CMS}^2 + \frac{\gamma^2 \beta^2}{(\gamma + 1)^2} p_{CMS}^2 \quad (19)$$

$$\langle \sin \alpha \cdot [G_1 - G_2] \rangle_\alpha = 2p_{CMS} \frac{\gamma}{\gamma + 1} \beta_y \quad (20)$$

Result (18) can be rewritten directly in the form of Eq.(7).

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